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The frequency spectrum of phonon emission from a heated two-dimensional electron gas in a strong magnetic field

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Abstract. The distribution of frequencies of phonons emitted from a 2DEG in a strong magnetic field that is heated relative to the lattice, for example by the passage of a current, is calculated using the self-consistent Born approximation to treat the disorder in the system. Results are shown for a range of temperatures and Landau level filling factors.

1. Introduction

Experiments are in progress [1, 2] that give information on the frequency distribution of phonons emitted by a two-dimensional electron gas (2DEG) in a strong magnetic field formed at a Si/SiO_2 interface in a MOSFET device which is heated by the passage of a current. The experiments utilize the fact that silicon is transparent to phonons below a critical frequency, ω_{iso}, for isotope scattering to occur and effectively opaque to those above it. The power transmitted ballistically is measured as a function of magnetic field or electron density with the total power supplied to the device held constant.

A previous paper [3] studied the angular distribution of phonons and total power dissipated by a heated 2DEG using a sequence of approximations for the treatment of the disorder in the electronic system and the purpose of this paper is to employ the same formalism for the calculation of the frequency spectrum and the relative contribution of different pairs of Landau levels. The rest of the paper is arranged as follows: in section 2 the frequency distribution function and the partial power due to a given pair of levels are defined in terms of the quantities discussed in [2], in section 3 the quantities are calculated for the case of ideal Landau levels while in section 4 they are evaluated in the self-consistent Born approximation [3]. Section 5 attempts to use the SCBA to predict the form of the field dependence in the experiments [1].

2. The frequency distribution function

In [3] the power emitted into phonon mode (s, Q) was expressed as

$$P_s(Q) = \Phi_s(Q)\Gamma(\omega_s(Q), q)$$

where Φ contains only information on the lattice and the material-dependent details of the electron–phonon coupling and the lowest sub-band wavefunction of the

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quantum well in which the 2DEG is formed and Γ is a structure function containing information on the states of the 2DEG only and $q = (q_x, q_y)$. The power emitted into phonon modes in unit frequency range about ω with polarization s is

$$D_s(\omega) = \int_{\mathrm{BZ}} \mathrm{d}^3Q\; P_s(Q)\delta(\omega - \omega_s(Q))$$

Employing the isotropic Debye approximation, $\omega_s(Q) = v_s Q$, so that Φ only depends on $Q = |Q|$ and $\theta = \arccos(Q \cdot \hat{n})$ where $\hat{n}$ is a unit vector normal to the 2DEG , gives the following expression for the frequency distribution function:

$$D_s(\omega) = 2\pi\left(\frac{\omega}{v_s}\right)^2 \int_0^{\pi} \mathrm{d}\theta\; \sin\theta\; \Phi_s(\omega/v_s, \theta)\Gamma(\omega, \omega\sin\theta/v_s).$$

The total power emitted is then

$$r = \sum_s \int_0^{\infty} \mathrm{d}\omega\; D_s(\omega).$$

In the strong-field limit the energy scale describing the disorder (i.e. the Landau level broadening) is much smaller than the energy scale associated with the magnetic field (the cyclotron energy $\epsilon_c = \hbar\omega_c$) and so the Landau level index is a good quantum number for the disordered system; hence the total power can be expressed as

$$r = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} r_{n,m}$$

where $r_{n,m}$ is the partial power due to electronic transitions from the $(n+m)$th Landau level down to the nth. It is these two quantities that will be discussed in the following. Magnetic units in which $\omega_c = l_c = \hbar = 1$ will be used throughout.

3. Emission from ideal Landau levels

From [3] it can be seen that in the absence of disorder

$$P_s(Q,\theta) = \Phi_s(Q,\theta)\frac{1}{2\pi}\sum_m \delta(\omega - m)\sum_n U_{n,m}(Q^2\sin\theta/2)f(\epsilon^0_{n+m})(1 - f(\epsilon^0_n))$$

where U is a matrix element factor which can be expressed in terms of Laguerre polynomials and $f(E)$ is the Fermi–Dirac distribution function. Hence $D_s(\omega) = \sum_m \delta(\omega - m)\Delta^m_s$ where

$$\Delta^m_s = \sum_n f(\epsilon^0_{n+m})(1 - f(\epsilon^0_n))\left(\frac{\omega}{v_s}\right)^2 \times \int_0^{\pi} \mathrm{d}\theta\; \sin\theta\; \Phi_s(\omega/v_s, \theta)U_{n,m}(\omega^2\sin^2\theta/2v_s^2).$$

This form of D makes it easy to pick out the contributions to $r_{n,m}$ which are shown in figures 1 and 2 normalized by the total power r for the case of TA phonon emission from a 2DEG in silicon at two different temperatures and chemical potentials.

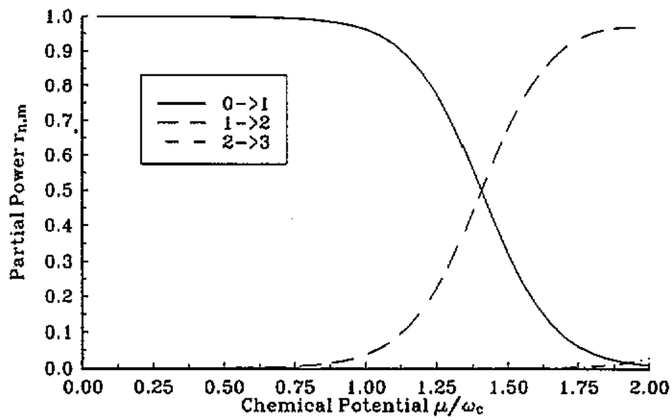

Figure 1. The power contribution from transitions between given Landau levels as a function of chemical potential, for $k_B T_e = \hbar\omega_c/8$.

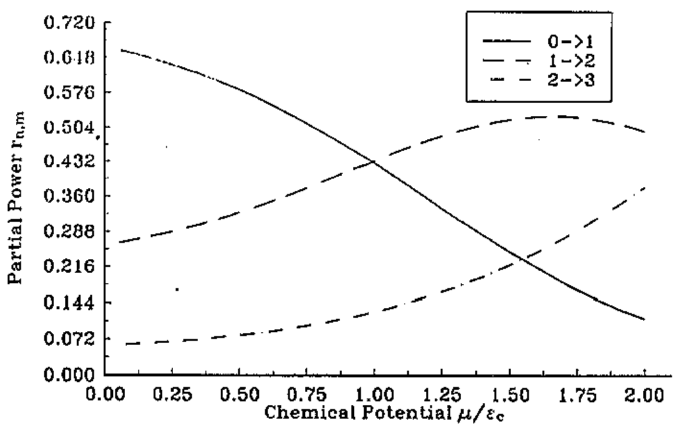

Figure 2. The power contribution from transitions between given Landau levels as a function of chemical potential, for $k_B T_e = \hbar\omega_c/2$.

4. Emission from a disordered 2DEG in a strong magnetic field

As shown in [2] the structure function Γ is related to the two-particle spectral function

$$S(q; E, \omega) = (1/\Omega)\overline{\mathrm{tr}\{\{\delta(E+\omega-\mathcal{H})\mathrm{e}^{\mathrm{i}q\cdot r}\delta(E-\mathcal{H})\mathrm{e}^{-\mathrm{i}q\cdot r}\}}$$

describing eigenfunction correlations by

$$\Gamma(\omega, q) = \int \mathrm{d}E\; f(E+\omega)\big(1-f(E)\big)S(q; E, \omega)$$

where Ω is the area of the 2DEG and the overbar denotes configurational averaging over the random potential in the Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + V(r) = (1/2m^*)[p - eA(r)]^2 + V(r)$$

the trace being taken over all single-particle states in the 2DEG. Assuming a Gaussian white-noise distribution for the random potential

$$\overline{V(r)} = 0 \qquad \overline{V(r)V(r')} = \tfrac{1}{2}\pi_c^2\nu^2$$

and using the SCBA form for $S(q; E, \omega)$ allows the evaluation of $D_s(\omega)$ and $r_{n,m}$. Figures 3 and 4 show $D(\omega)$ for two choices of temperatures and chemical potentials. Figures 5 and 6 show the variation of $r_{n,m}/r$ with filling factor (electron density normalized to the number of states per Landau level) for the transitions that contribute appreciably for two different values of the temperature, again for the case of TA emission from silicon.

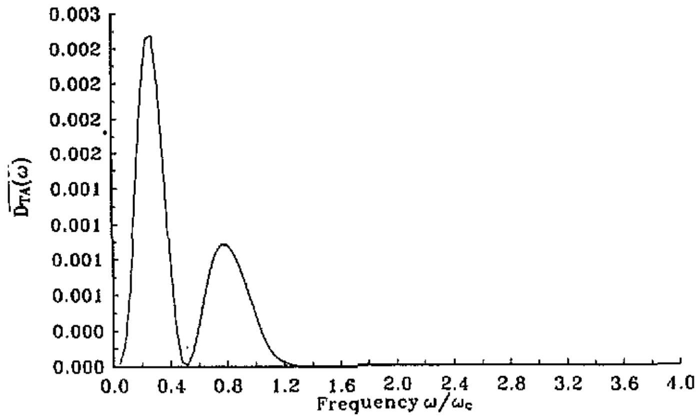

Figure 3. The frequency spectrum of TA phonons emitted by a heated silicon 2DEG at $\mu = \hbar\omega_c/2$ and $k_B T_e = \hbar\omega_c/8$.

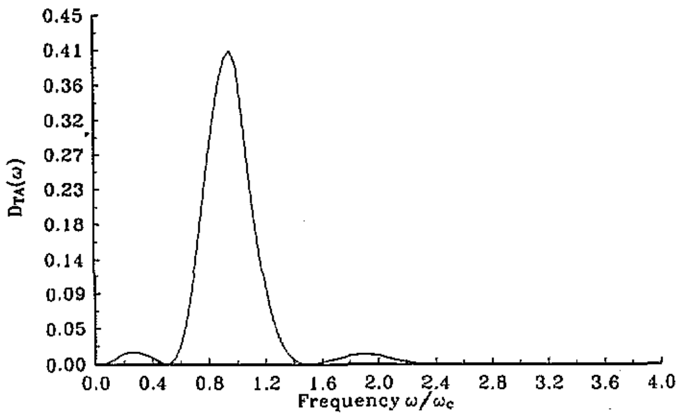

Figure 4. The frequency spectrum of TA phonons emitted by a heated silicon 2DEG at $\mu = \hbar\omega_c$ and $k_B T_e = \hbar\omega_c/2$

5. Field dependence of the ballistic fraction

As discussed in the introduction, experiments [1] detect the power emitted by a heated 2DEG as ballistically transmitted phonons. As the magnetic field, B, is varied the ratio (in conventional units) ω_{iso}/ω_c varies as $1/\sqrt{(B)}$, hence by sweeping the

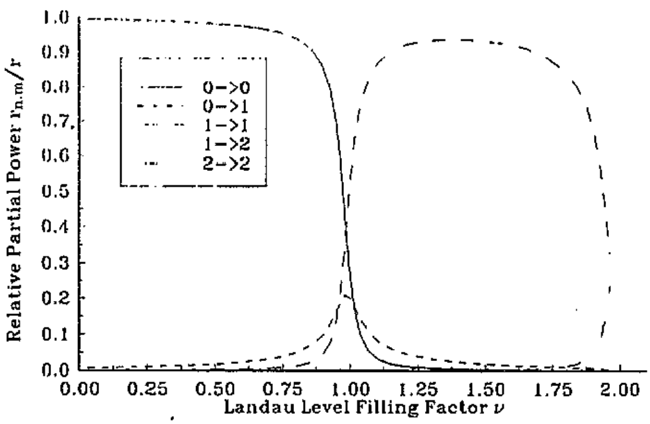

Figure 5. TA phonon power contribution from transitions between given Landau levels as a function of chemical potential $k_B T_e = \hbar\omega_c/8$, in silicon

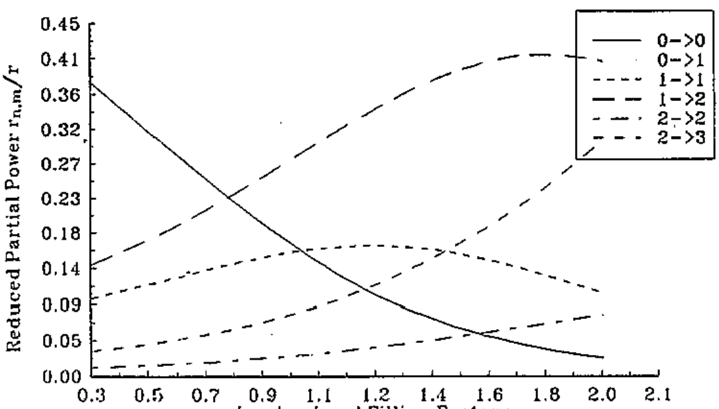

Figure 6. TA phonon power contribution from transitions between given Landau levels as a function of chemical potential $k_B T_e = \hbar\omega_c/2$ in silicon

field the effective value of the cutoff frequency is varied. The fraction of the phonon power which is transmitted ballistically can be expressed as

$$\alpha(\omega_{\text{iso}}) = \sum_s \int_0^{\omega_{\text{iso}}} \mathrm{d}\omega\ D_s(\omega) \Big/ \sum_s \int_0^{\infty} \mathrm{d}\omega\ D_s(\omega).$$

Numerical integration of the forms given previously for D lead to the magnetic field dependence shown in figure 7. As would be expected there is a cut-off field $B_c = m^*\omega_{\text{iso}}/e$ below which $\omega_{\text{iso}} > \omega_c$, so cyclotron phonons (those coming from $m = 1$ inter-Landau-level transitions) which carry away most of the power from the 2DEG are transmitted ballistically. At higher fields only the low-energy phonons from intra-Landau-level transitions are transmitted and the ballistic fraction α drops markedly. It is hoped that detailed comparison of the theoretical predictions with the experiments will lead to a means of assessing the form of the density of states in the devices.

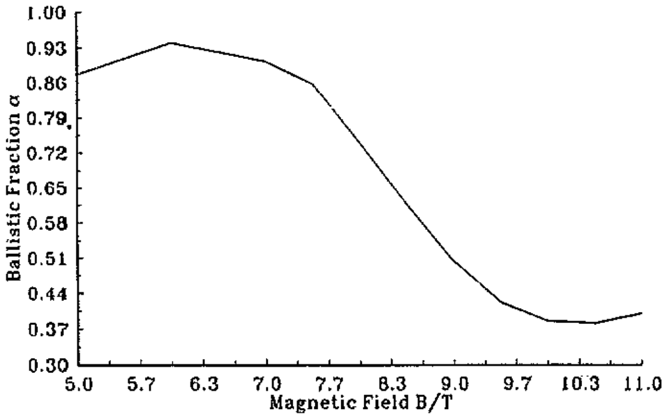

Figure 7. The magnetic field dependence of the fraction of the total power supplied to a silicon 2DEG which is emitted as ballistically propagating phonons. T_e=75 K; electron density is $n_s = 1.1 \times 1.1^{16}$ m^{-3}.

6. Summary

The theoretical method described in [3] has been extended to the calculation of the frequency spectrum of phonons emitted by a heated 2DEG in a strong magnetic field and related quantities including one that is directly accessible to experiment.

References

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